Solar Quadrupole moment and Purely Relativistic Gravitation Contributions

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Abstract: The expansion in multipoles J(*l*, *l* = 2, ..), of the gravitational potential of a rotating body affects the orbital motion of planets at a relativistic level. We will recall here the recent progresses made in testing General Relativity through the contribution of the solar quadrupole moment. Using the Eddington-Robertson parameters, we recall the constraints both on a theoretical and experimental point of view. Together with γ, which encodes the amount of curvature of space-time per unit rest-mass, the post-Newtonian parameter beta contributes to the relativistic precession of planets. The latter parameter encodes the amount of non-linearity in the superposition law of gravitation. We will show that it is still difficult to decorrelate J(*l*=2), γ and β. Moreover, all the planetary dynamics-based values are biased by the Lense Thiring effect, which has never been modelled and solved-for so far, but can be estimated to 7%. It is thus possible to get a good estimate of the solar quadrupole moment: $1.82 \times 10^{-7} \leq J2 \leq 2.16 \times 10^{-7}$.

Introduction

Of all the solar fundamental parameters (mass, diameter, gravity at surface, angular momentum…), the gravitational moments have been ignored in the past, mainly due to the great difficulty to get a reliable estimate. Even though the order of magnitude of the solar quadrupole moment J_2 is now known to be 10^{-7}, its precise value is still discussed. Furthermore, stellar equations combined with a differential rotation model, as well as the inversion techniques applied to helioseismology, are methods which are solar model dependent, i.e. implying solar density and rotation laws. Hence the need for dynamical estimates of the solar quadrupole moment, based on the motion of spacecrafts, celestial bodies or light in the gravitational field of the Sun.

Theoretical background

The space-time geometry around a spherical star is described, in isotropic coordinates, by

$$ds^2 = -\left(1 - 2\frac{GM}{c^2 r} + 2\left(\frac{GM}{c^2 r}\right)^2\right)(cdt)^2 + \left(1 + 2\frac{GM}{c^2 r}\right)[dx^2 + dy^2 + dz^2]$$

where *G* is the gravitational constant, *c* the speed of light and *M* the mass of the star.

The Parameterized Post-Newtonian (PPN) formalism generalizes the above metric by taking into account ten parameters, of which β and γ are the most important, so that

$$ds^2 = -\left(1 - 2\frac{GM}{c^2 r} + 2\beta\left(\frac{GM}{c^2 r}\right)^2\right)(cdt)^2 + \left(1 + 2\gamma\frac{GM}{c^2 r}\right)[dx^2 + dy^2 + dz^2]$$

γ encodes the amount of curvature produced by unit rest mass, and is tested by deflection of light. β encodes the degree of non-linearity in the superposition law for gravity. In GR, $\beta = \gamma = 1$. From the PPN formalism, the perihelion precession of a planet, in the vicinity of the Sun becomes:

$$\delta\phi = \frac{6\pi GM_\odot}{a(1-e^2)c^2}\frac{(2-\beta+2\gamma)}{3}$$

where a is the semi-major axis, e the eccentricity, ϕ the true anomaly. Under the influence of an oblate Sun, the perihelion shift becomes:

$$\delta\phi = \frac{6\pi GM_\odot}{a(1-e^2)c^2}\left[\frac{(2-\beta+2\gamma)}{3}\right] + \frac{6\pi}{2}R_\odot^2\frac{(1-3/2\sin^2 i)}{a^2(1-e^2)^2}J_2$$

with i the orbital inclination with respect to the solar equator, and R is the solar radius. Lastly, a correction must be applied, the Lense-Thirring effect, due to the rotation of the central mass, which produces a secular precession of the longitude of the ascending node. It turns out that accurately measuring the perihelion shift of planets, combined with the most modern γ and β values, would lead to a precise independent estimate of J_2. Note that in pure GR, the first term of the above equation is 42''.9794. Today the best determinations of β and γ are:

$(\gamma-1) \times (10^{-5}) = 2.1 \pm 2.3$ (Cassini mission)
$(\beta-1) \times (10^{-4}) = 1.2 \pm 1.1$ (Lunar laser ranging)

Constraints from Planetary Orbital Motions

We processed ephemerids tests, starting epoch Julian day 2440400.5, carried out by removing or including the motion of Mercury, Venus and Mars to check the correlation between J_2, β and γ. It was found a strong correlation: about 45% between J_2 and β, -56% between J_2 and γ and 30% between β and γ; the nominal GR solution gives J_2= (2.2 +/- 0.4) x 10^{-7} ($\beta = \gamma = 1$). The only way to decorrelate is to increase the number of planets (and asteroids) taken into consideration. Improvements from numerical Ephemerides of Planets and the Moon (EPM), lead to the following results (Pitjeva and Pitjev, 2014), after correction of the Lense-Thirring (LT) effect:

Parameter	EPM2004	EPM2008	EPM2011	EPM2013
$J_2 \cdot 10^{-7}$	**1,8 $\pm$ 0,6**	**1,92 $\pm$ 0,57**	**1,89 $\pm$ 0,38**	**2,06 $\pm$ 0,23**

On another hand, the Planetary and Lunar Ephemerides DE430 and DE431 (Folkner et al., 2014) generated by fitting numerically orbits of the Moon, our 9 Planets and 1467 asteroids lead to 2.11 x 10^{-7} in pure GR ($\gamma = \beta = 1$), i.e. 1.96 x 10^{-7} after LT correction.

Conclusion

It has been shown since a long time that a gravitational quadrupole moment of the Sun influences the motion and inclination of the planetary orbits. Up to the beginning of the year 2014, the range of J(*l*=2) was set up from 1.4 x 10^{-7} to 2.4 x 10^{-7}. Due to the progress in ephemerides computations, today, one can restrict J(*l*=2) to **1.99 x 10^{-7}** with an error of **± 0.17 x 10^{-7}**.

With new ground-based observations using radar astrometry (at Arecibo by Margot et al.) in the inner solar system, scheduled to the next 3 years, it is expected to provide a purely dynamical measurement of J_2 at the 10^{-8} level. The same conclusion arises with the incoming GAIA data, which will provide an unprecedented number of observations of perihelion shifts.

However, the temporal variation of this parameter is not definitely known either, even if constraints can be put.